## Learning Objectives

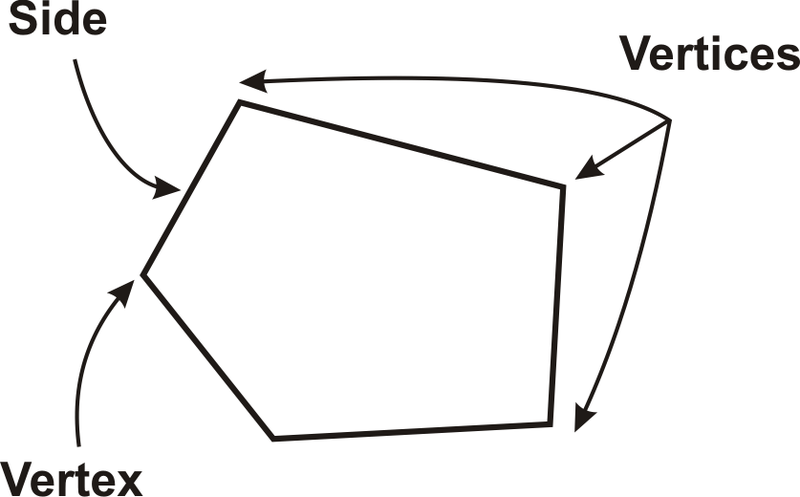
* Define polygon.
* Identify polygons as convex or concave.
* Classify polygons by number of sides.
* Determine if a polygon is regular or not.

## Defining Polygons

A **polygon** is any closed planar figure that is made entirely of line segments that intersect at their endpoints. Polygons can have three or more sides and angles.

* A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a closed figure made of straight line segments that intersect at their endpoints.
* **Polygons** can have \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or more sides and angles.

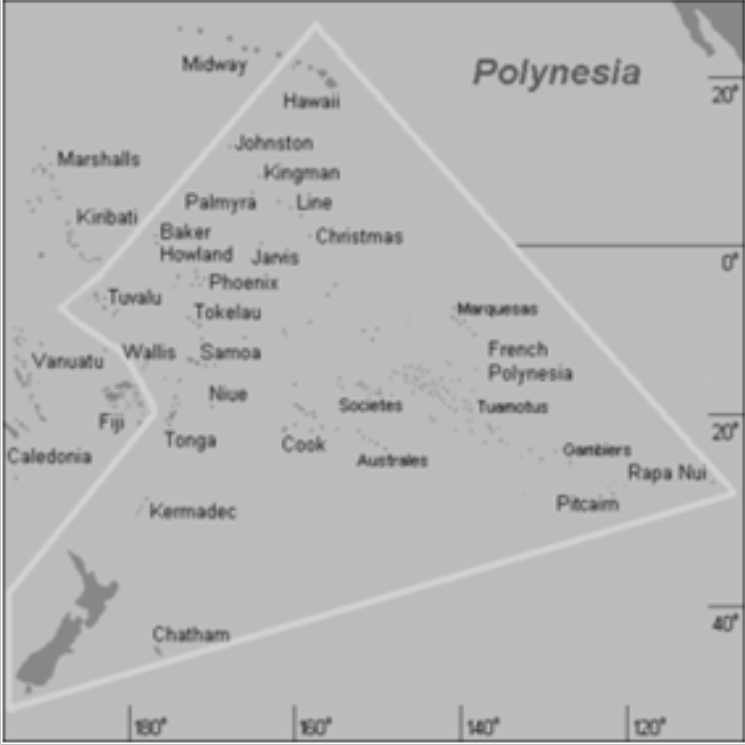
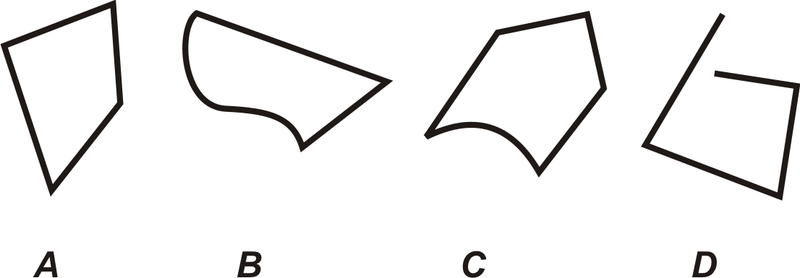
The segments are called the **sides** of the polygons, and the points where the segments intersect are called **vertices**. [Note that the singular of vertices is **vertex**.]

The following shape to the right is a **polygon**:

* The segments in a **polygon** are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Points where the **sides** of a **polygon** intersect are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* The plural form of **vertex** is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**The prefix “poly”** means “many.”

For example, the region of the world known as “Polynesia” is made of many islands.

**Example 1:** Which of the figures to the right is a polygon?  
(check next page for answers)

The easiest way to identify the polygon is to identify which shapes are not polygons.

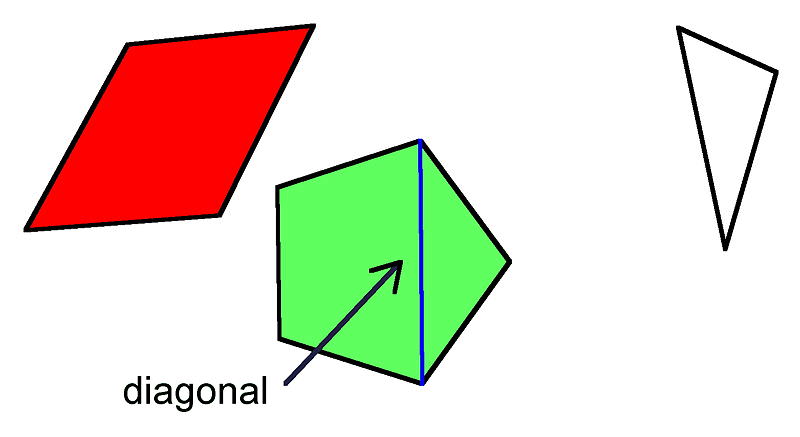
Choices **B** and **C** each have at least one curved side. So they cannot be polygons because polygons must have straight sides.

Choice **D** has all straight sides, but one of the vertices is not at the endpoints of the two adjacent sides, so it is not a polygon.

Choice **A** is composed entirely of line segments that intersect at their endpoints. So, it is a **polygon**. The correct answer is **A**.

## Diagonals

Lines segments that connect to vertices traveling only on the interior of the shape are called **diagonals**.



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Classifying Polygons by their Numbers of Sides

The most common way to classify a **polygon** is by the number of sides. Regardless of whether the polygon is convex or concave, it can be named by the number of sides.

The prefix in each name reveals the number of sides. Refer to the polygon chart at the end of this lesson to name and classify samples of polygons.

## Regular, Equiangular and Equilateral Polygons

Polygons that have **congruent sides** are equilateral.

Polygons that have **congruent angles** are equiangular.

If a polygon is both equilateral and equiangular, it is called a **regular** polygon.

A square is an example of a **regular** polygon because it has four congruent angles and four congruent sides.

* **Equilateral polygons** have congruent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* **Equiangular polygons** have congruent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* A **regular polygon** is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

You can probably figure out that “equi” means “equal” and “angular” means “angles.”

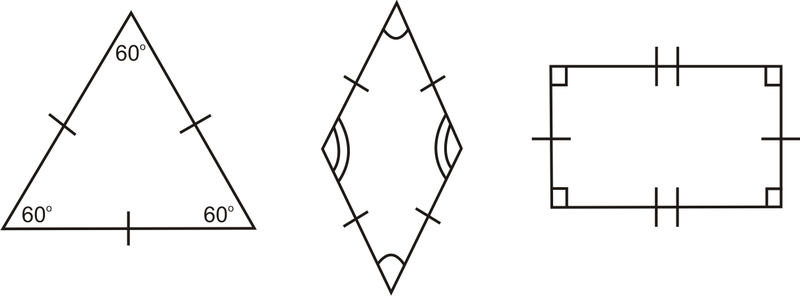
But what about “lateral”? Lateral is related to the Spanish word “lado,” which means “side.”

What do you think the word “quadrilateral” means when you break it down?

“Regular” in this case means “consistent,” as in “the song has a regular rhythm” or “her regular bedtime is 10:00.”

**Reading Check:**

Identify each polygon below as equiangular, equilateral, or regular.



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Graphic Organizer for Lesson 9

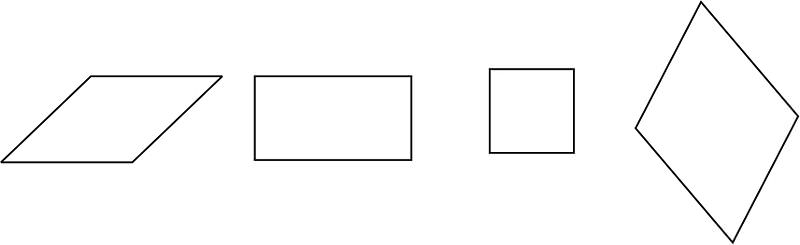
**Classifying Polygons by their Numbers of Sides**

The most common way to classify a polygon is by the number of sides. Regardless of whether the polygon is convex or concave, it can be named by the number of sides. The prefix in each name reveals the number of sides. The chart below shows names and samples of polygons.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Polygon Name** | **Number of Sides** | **Sample Drawings** | **Draw your own example** | **Can you think of any other words that begin with...** |
| ***Triangle*** | 3 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A3470032283c5cfa4788fdcf80556ddb4c693b082453667b515f4614e%2BIMAGE_TINY%2BIMAGE_TINY.1 |  | ..tri?   * Tricycle * Triple |
| ***Quadrilateral*** | 4 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A05283ac64953ab9046739c3704e066417b0de35f41d098c439ee08a4%2BIMAGE_TINY%2BIMAGE_TINY.1 |  | ...qua?   * Quarter * Quadruple |
| ***Pentagon*** | 5 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3Ac4d621ba03cd535d46c4fa98818e2ca3bc6fe2414393845314dffbc5%2BIMAGE_TINY%2BIMAGE_TINY.1 |  | ...penta?   * Pentameter |
| ***Hexagon*** | 6 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A5ed062ed0a85a61873cc6a6ec0b46f0f664932d8a028f7bf60acb56f%2BIMAGE_TINY%2BIMAGE_TINY.1 |  |  |
| ***Heptagon*** | 7 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A9415c79a95c400d041531cba8ca9f2baad0f43c0174a02e588be410a%2BIMAGE_TINY%2BIMAGE_TINY.1 |  |  |
| ***Octagon*** | 8 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A02f243448fe0d7a6edaba63dafbe08f48edae8fe0edb1d6dc26cedfe%2BIMAGE_TINY%2BIMAGE_TINY.1 |  | ...octo?   * Octopus |
| ***Nonagon*** | 9 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A7e1ce4c81a2b151af8410ba72e1debe65d540f7a5c6bf098e8f4348c%2BIMAGE_TINY%2BIMAGE_TINY.1 |  |  |
| ***Decagon*** | 10 | https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A8a15bd6e1c8f3da2f692e32abec4028a017007bf52d46bfdc786a2ec%2BIMAGE_TINY%2BIMAGE_TINY.1 |  | ...deca?   * Decade |

**SECTION 2: Parallelograms**   
**Learning Objectives**

* Identify properties of parallelograms.
* Describe the relationships between opposite sides in a parallelogram.
* Describe the relationship between opposite angles in a parallelogram.
* Describe the relationship between consecutive angles in a parallelogram.
* Describe the relationship between the two diagonals in a parallelogram.
* Apply parallelogram properties to solve problems.

**Parallelograms**A **parallelogram** is a **quadrilateral** with two pairs of *parallel* sides. Each of the shapes shown to the right is a parallelogram:

*Do you remember what a* ***quadrilateral*** *is?*

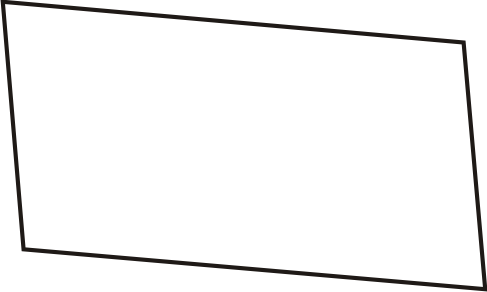
*It is a polygon with four sides.*

*Some other words that have the same prefix as* ***quadrilateral*** *are:*

* ***quad****ruple – to multiply by 4*
* ***qua****rter – one fourth*
* ***quad****ruplets – four brothers and sisters born at the same time*

As you can see, **parallelograms** come in a variety of shapes. The *only* defining feature is that *opposite sides are parallel*. But, once we know that a figure is a parallelogram, we have very useful theorems we can use to solve problems involving parallelograms.  
  
A **parallelogram** is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ with two pairs of *parallel* sides.

A **quadrilateral** is a polygon with \_\_\_\_\_\_\_\_\_\_\_\_ sides.

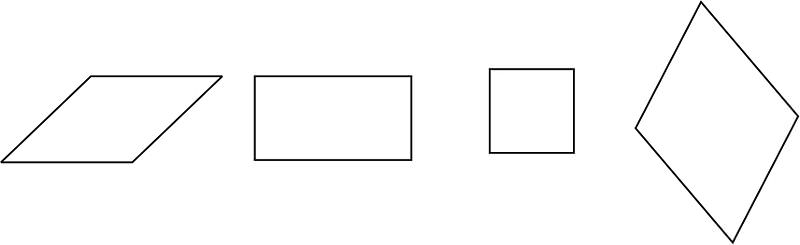
**Reading Check:**

1. What makes a quadrilateral a parallelogram?

2. Mark the parallel lines with arrows ( > and >> ) to show the pairs of *parallel* lines in the picture below:

**Opposite Sides in a Parallelogram**

There are many types of parallelograms. Opposite sides are always parallel.



One of the most important things to know, however, is that *opposite sides* in a parallelogram are also *congruent*.

**Opposite Sides of Parallelogram Theorem**

The opposite sides of a parallelogram are congruent.

Look at the pair of parallel line segments here:

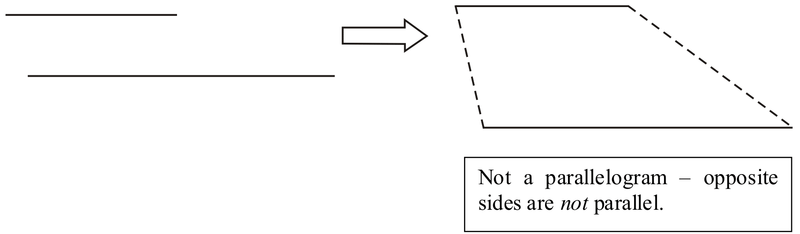
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If you want to connect the endpoints of the line segments, you must draw in two **parallel, congruent** line segments.



If the line segments are not congruent, you cannot make a parallelogram:



So, even though parallelograms are *defined* by their parallel opposite sides, one of their *properties* is that opposite sides be *congruent*.

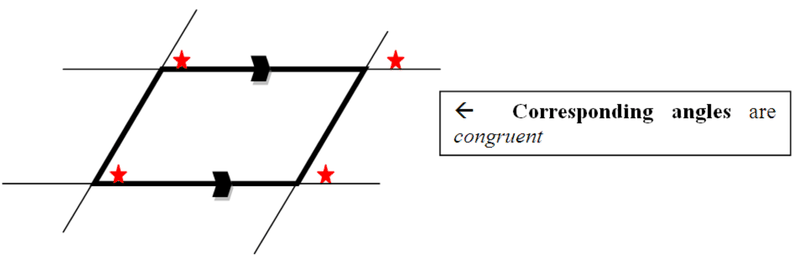
**Opposite Angles in a Parallelogram**

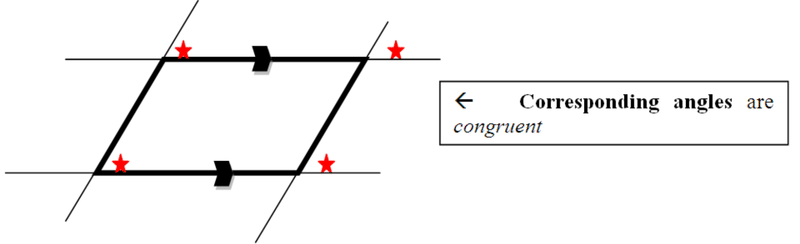
Not only are *opposite sides* in a parallelogram *congruent; opposite angles* are also *congruent*.

**Opposite Angles in Parallelogram Theorem**

The *opposite angles* of a parallelogram are *congruent*.

You have learned that when lines are *parallel*, their **corresponding angles** are *congruent*.

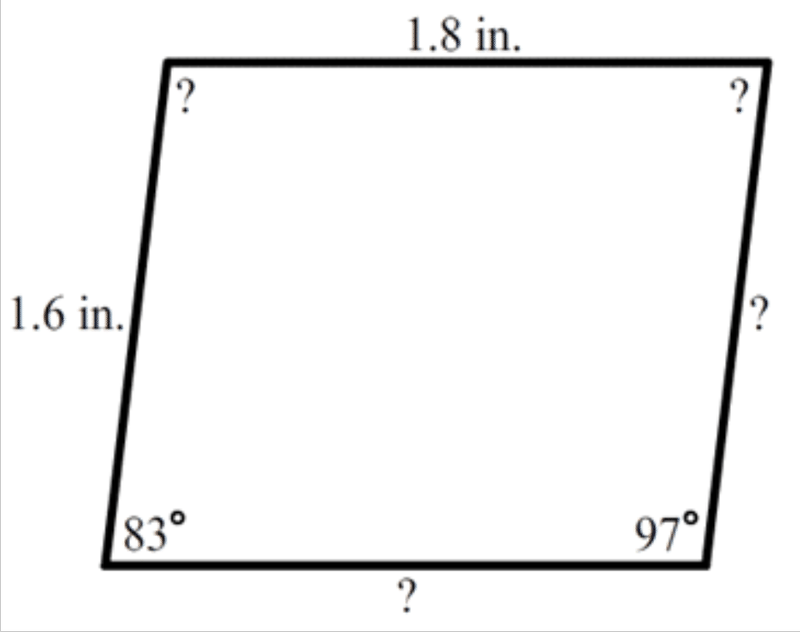




Opposite sides and opposite \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ in a parallelogram are *congruent*.

**Reading Check:**

*Mark as many measurements as you can in the picture of the parallelogram below.*



**Consecutive Angles in a Parallelogram**

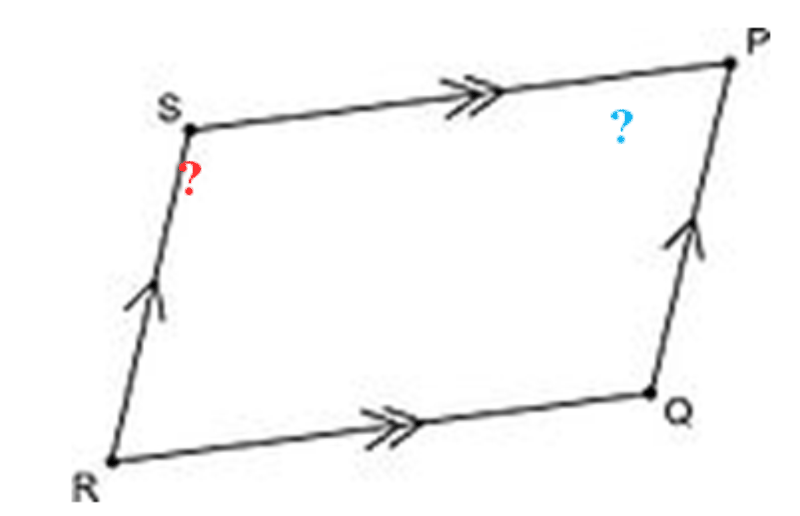
At this point, you understand the relationships between opposite sides and opposite angles in parallelograms.

In a **parallelogram...**

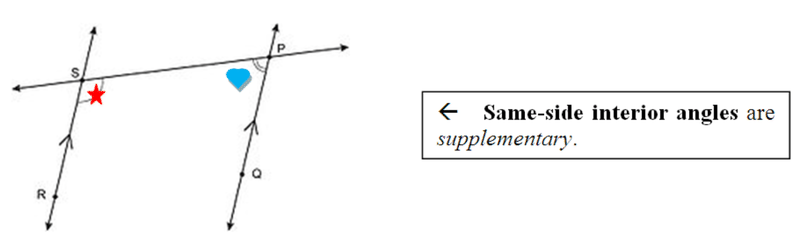
* Opposite sides are *parallel* (this is the definition of a parallelogram)
* Opposite sides are *congruent*
* Opposite angles are *congruent*

Think about the relationship between *consecutive* angles in a parallelogram. You have studied this scenario before, but you can apply what you have learned to parallelograms.

Examine the parallelogram below:



Imagine that you are trying to find the relationship between \angle SPQ and \angle PSR in the diagram on the previous page. To help you understand the relationship, extend all of the segments involved with these angles and remove \overline{RQ} like we have below:



What you should notice is that \overleftrightarrow{PQ} and \overleftrightarrow{SR} are two *parallel* lines cut by transversal \overleftrightarrow{SP}.

* \overleftrightarrow{PQ} and \overleftrightarrow{SR} are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ lines.
* \angle SPQ and \angle PSR are same-side \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ angles.

Earlier in this lesson, you learned that in this scenario, two **consecutive interior angles** are *supplementary*; they sum to 180^\circ. The same is true within the parallelogram. Any two **consecutive angles** inside a parallelogram are *supplementary*.

**Consecutive Angles in Parallelogram Theorem**

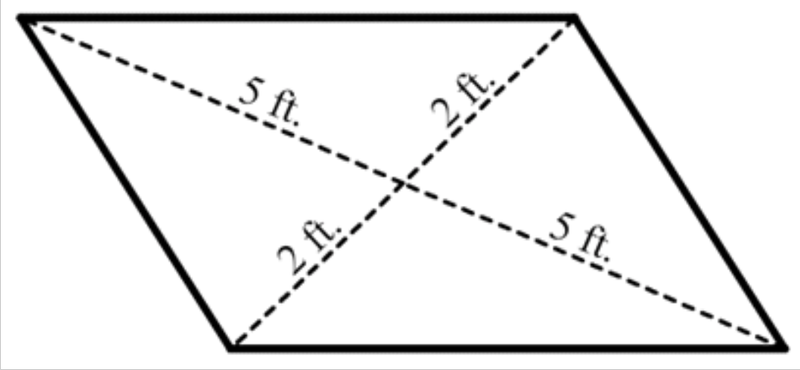
Any two *consecutive* angles of a parallelogram are *supplementary*.

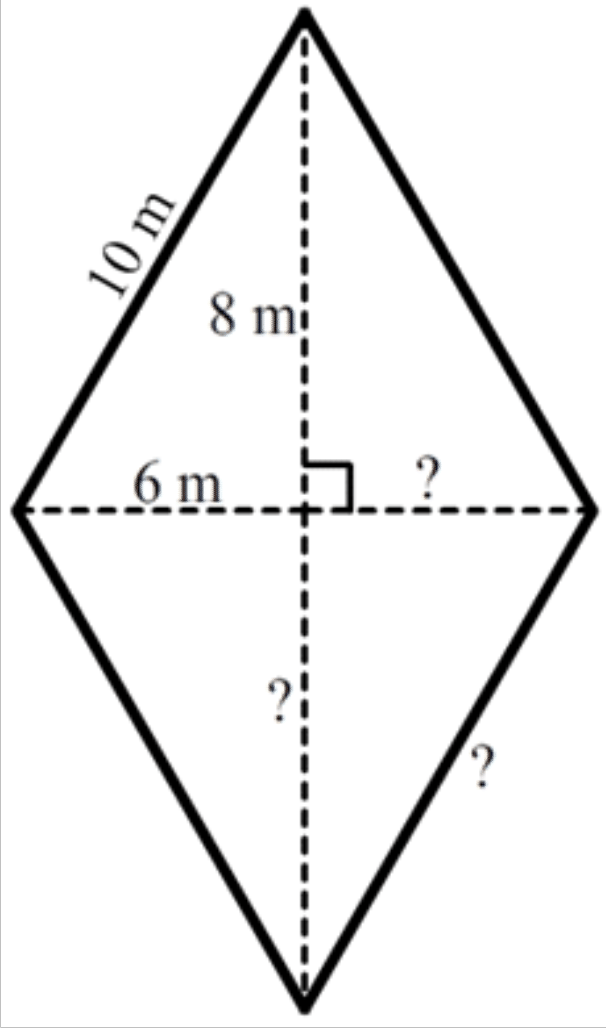
**Diagonals in a Parallelogram**

There is one more relationship to examine within parallelograms. When you draw the two **diagonals** inside parallelograms, they *bisect* each other. This can be very useful information for examining larger shapes that may include parallelograms.

**Diagonals in a Parallelogram Theorem**

The *diagonals* of a parallelogram *bisect* one another.



Bisecting diagonals means that the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ cut each other in half.

**Reading Check:**

1. *True or False:* Opposite angles in a parallelogram are congruent.

2. *True or False:* Consecutive angles in a parallelogram are congruent.

3. *True or False:* Opposite sides in a parallelogram are both congruent and parallel.

4. *Find the measure of the missing segments in the picture to the right 🡪*